History of Islam

An encyclopedia of Islamic history

Al Khwarizmi

Al-Khwarizmi-the Father of Algebra

Contributed by Prof. Dr. Ibrahim B. Syed

Abu Ja'far Muhammad ibn Musa al-Khwarizmi, (780 – 850 CE), was the grandfather of computer science and the father of Algebra. He was the popularizer of Arabic numerals, adopter of zero (the symbol, that is) and the decimal system, astronomer, cartographer, in briefs an encyclopedic scholar.

BAYT Al-HIKMA (House of Wisdom)

In the year 832, Caliph Al Ma'mun [b. Baghdad, 786, d. Tarsus, Cilicia, August 833] founded the "House of Wisdom" in Baghdad, a center for study and research similar to the earlier Museum in Alexandria. Its most famous scholars were the mathematicians Muhammad ibn Musa Al-Khwarizmi and the Banu Musa ("sons of Moses"), three brothers who directed the translation of Greek works from Antiquity. [(7)]

The modern word algorithm is derived from the name, al-Khwarizmi, the best mathematician of his age, thanks to his book, al-Kitab al-mukhtasar fi Hisab al-jabr w'al-muqabala, (a book showing how to solve equations and problems derived from ordinary life) which means "The Compendious Book on Calculation by Completion and Balancing", which later evolved into algebra, was the first written text on the subject. In al-Khwarizmi's time, algebra was a practical system for solving all kinds of problems "in cases of inheritance, contracts, surveying, tax collection, legacies, partition, lawsuits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computations, and other objects of various sorts and kinds are concerned." Al-jabr was about removing the negative terms from an equation, while al-muqabala meant "balancing" the values of an equation across an equals sign.

It is the title of this text that gives us the word "algebra". It is the first book to be written on algebra. In al-Khwarizmi's own words, the purpose of the book was to teach what was easiest and most useful in arithmetic, such as what was constantly required in cases of inheritance, legacies, partition, lawsuits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computations, and other objects of various sorts and kinds were concerned.

This does not sound like the contents of an algebra text, and indeed only the first part of the book is a discussion of what we would today recognize as algebra. However it is important to realize that the book was intended to be highly practical, and that algebra was introduced to solve real life problems that were part of everyday life in the Islamic empire at that time.

After introducing the natural numbers, he discusses the solution of equations. His equations are linear or quadratic and are composed of units (numbers), roots (x) and squares (x2). He first reduces an equation to one of 6 standard forms, using the operations of addition and subtraction, and then shows how to solve these standard types of equations. He uses both algebraic methods of solution and the geometric method of completing the square.

The next part of al-Khwarizmi's Algebra consists of applications and worked examples. He then goes on to look at rules for finding the area of figures such as the circle, and also finding the volume of solids such as the sphere, cone, and pyramid. This section on mensuration certainly has more in common with Hindu and Hebrew texts than it does with any Greek work. The final part of the book deals with the complicated Islamic rules for inheritance, but requires little from the earlier algebra beyond solving linear equations. (8)

TEXTBOOK OF ALGEBRA (9)

Each chapter was followed by geometrical demonstration and then many problems were worked out. Some of his problems were formal while others were in practical context. An example of his formal problem follows:

"If from a square I subtract four of its roots and then take one-third of the remainder, finding this equal to four of the roots, the square will be 256."

He explained it in the following manner:

"Since one-third of the remainder is equal to four roots, one knows that the remainder itself will equal 12 roots. Therefore, add this to the four, giving 16 roots. This (16) is the root of the square. The above can also be stated in terms of modern notation as 1/3 (x2 – 4x) = 4x."

Khwarizmi, in a chapter on commercial transactions, states that "mercantile transactions and all things pertaining thereto involve two ideas and four numbers." Karpinski in his translation of the book explains: The two ideas appear to be the notions of quantity and cost; the four numbers represent unit of measure and price per unit, quantity desired and cost of the same.

An example of Al-Khwarizmi's mercantile problem: (9)

"A man is hired to work in a vineyard 30 days for 10 dollars. He works six days. How much of the agreed price should he receive?"

Explanation: "It is evident that since days are one-fifth of the whole time; and it is also evident that the man should receive pay having the same relation to the agreed price that the time he works bears to the whole time, 30 days. What we have proposed, is explained as follows. The month, i.e., 30 days, represents the measure, and ten represents the price. Six days represents the quantity, and in asking what part of the agreed price is due to the worker you ask the cost. Therefore multiply the price 10 by the quantity 6, which is inversely proportional to it. Divide the product 60 by the measure 30, giving 2 Dollars. This will be the cost, and will represent the amount due to the worker."

The text book of Algebra was intended to be highly practical and it was introduced to solve real life problems that were part of everyday life in the Islamic world at that time. Early in the book al-Khwarizmi wrote:-

"When I consider what people generally want in calculating, I found that it always is a number. I also observed that every number is composed of units, and that any number may be divided into units. Moreover, I found that every number which may be expressed from one to ten, surpasses the preceding by one unit: afterwards the ten is doubled or tripled just as before the units were: thus arise twenty, thirty, etc. until a hundred: then the hundred is doubled and tripled in the same manner as the units and the tens, up to a thousand; … so forth to the utmost limit of numeration."[10]

SOLUTIONS OF EQUATIONS [11]

Having introduced the natural numbers, al-Khwarizmi introduces the main topic of this first section of his book, namely the solution of equations. His equations are linear or quadratic and are composed of units, roots and squares. For example, to al-Khwarizmi a unit was a number, a root was x, and a square was x2. However, although we shall use the now familiar algebraic notation in this article to help the reader understand the notions, Al-Khwarizmi's mathematics is done entirely in words with no symbols being used.

He first reduces an equation (linear or quadratic) to one of six standard forms:

1. Squares equal to roots. Example: $ax2 = bx$
2. Squares equal to numbers. Example: $ax2 = b$
3. Roots equal to numbers. Example: $ax = b$
4. Squares and roots equal to numbers. Example: $ax2 + bx = c$ e.g. $x2 + 10 x = 39$
5. Squares and numbers equal to roots. Example: $ax2 + c = bx$ e.g. $x2 + 21 = 10 x$
6. Roots and numbers equal to squares. Example: $ax2 = bx + c$, e.g. $3 x + 4 = x2$

The reduction is carried out using the two operations of al-jabr and al-muqabala. Here "al-jabr" means "completion" and is the process of removing negative terms from an equation. For example, using one of al-Khwarizmi's own examples, "al-jabr" transforms $x2 = 40 x – 4 x2$ into $5 x2 = 40 x$. The term "al-muqabala" means "balancing" and is the process of reducing positive terms of the same power when they occur on both sides of an equation. For example, two applications of "al-muqabala" reduces $50 + 3 x + x2 = 29 + 10 x$ to $21 + x2 = 7 x$ (one application to deal with the numbers and a second to deal with the roots).

Al-Khwarizmi then shows how to solve the six standard types of equations. He uses both algebraic methods of solution and geometric methods. For example to solve the equation $x2 + 10 x = 39$ he writes:-

… Square and 10 roots are equal to 39 units. The question therefore in this type of equation is about as follows: what is the square which combined with ten of its roots will give a sum total of 39? The manner of solving this type of equation is to take one-half of the roots just mentioned. Now the roots in the problem before us are 10. Therefore take 5, which multiplied by itself, gives 25, and an amount which you add to 39 giving 64. Having taken then the square root of this which is 8, subtract from it half the roots, 5 leaving 3. The number three therefore represents one root of this square, which itself, of course is 9. Nine therefore gives the square.

The geometric proof by completing the square follows. Al-Khwarizmi starts with a square of side x, which therefore represents x2 (Figure 1). To the square we must add 10x and this is done by adding four rectangles each of breadth 10/4 and length x to the square (Figure 2). Figure 2 has area $x2 + 10 x$ which is equal to 39. We now complete the square by adding the four little squares each of area. $5/2* 5/2 = 25/4$.

Hence the outside square in Fig 3 has area 4 25/4 + 39 = 25 + 39 = 64. The side of the square is therefore 8. But the side is of length 5/2 + x + 5/2 so x + 5 = 8, giving x = 3.

Al-Khwarizmi continues his study of algebra in Hisab al-jabr w'al-muqabala by examining how the laws of arithmetic extend to arithmetic for his algebraic objects. For example he shows how to multiply out expressions such as (a + b x) (c + d x).

although again we should emphasize that al-Khwarizmi uses only words to describe his expressions, and no symbols are used. The scientific historian, Roshdi Rashed 12 writes:-

Al-Khwarizmi's concept of algebra can now be grasped with greater precision: it concerns the theory of linear and quadratic equations with a single unknown and the elementary arithmetic of relative binomials and trinomials. … The solution had to be general and calculable at the same time and in a mathematical fashion, that is, geometrically founded. … The restriction of degree, as well as that of the number of unsophisticated terms, is instantly explained. From its true emergence, algebra can be seen as a theory of equations solved by means of radicals, and of algebraic calculations on related expressions…

If this interpretation is correct, then al-Khwarizmi was as Sarton [12] writes:-… the greatest mathematician of the time, and if one takes all the circumstances into account, one of the greatest of all time….

Al-Khwarizmi also wrote a treatise on Hindu-Arabic numerals. The Arabic text is lost but a Latin translation, Algoritmi de numero Indorum in English Al-Khwarizmi on the Hindu Art of Reckoning gave rise to the word algorithm deriving from his name in the title as mentioned earlier. Unfortunately the Latin translation (translated into English) is known to be much changed from al-Khwarizmi's original text (of which even the title is unknown). The work describes the Hindu place-value system of numerals based on 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. The first use of zero as a place holder in positional base notation was probably due to al-Khwarizmi in this work. Methods for arithmetical calculation are given, and a method to find square roots is known to have been in the Arabic original although it is missing from the Latin version. [12]

… the decimal place-value system was a fairly recent arrival from India and … al-Khwarizmi's work was the first to expound it systematically. Thus, although elementary, it was of seminal importance.

Khwarizmi developed detailed trigonometric tables containing the sine functions which later included tangent functions. Khwarizmi's book on arithmetic was translated into Latin and published in Rome in 1857 by Prince Baldassare Boncompagni and appears as part 1 of a volume entitled Tratti d' aritmetica. The book is titled as Algorithmi de numero indorum which means "Khwarizmi concerning the Hindu art of reckoning." Many of his books were translated into Latin and used as a principle mathematical text book in European universities until 16th century. Among them these two books had important place: Kitab al-Jama wal-Tafreeq bil Hisab al-Hindi and Kitab al-Jabr wa al-muqabala.

Khwarizmi's contribution and influence are tremendous. Two important books on arithmetic, Carmen de Algorismo and Algorismus vulgaris which were written in 12th and 13th century respectively owe a lot to the Khwarizmi's book and were used for several hundred years in Europe. Abu Kamil whose work on mathematics was based on Khwarizmi's works kept the influence of Khwarizmi on Leonardo of Pisa, a 13th century scholar and up to Middle Ages and during the Renaissance. [9]

ASTRONOMY [13]

Al Khwarizmi also wrote an important work on astronomy, covering calendars, calculating true positions of the sun, moon and planets, tables of sines and tangents, spherical astronomy, astrological tables, parallax and eclipse calculations, and visibility of the moon. Although his astronomical work is based on that of the Indians, and most of the values from which he constructed his tables came from Hindu astronomers, al-Khwarizmi must have been influenced by Ptolemy's work too. Al-Khwarizmi performed detailed calculations of the positions of the Sun, Moon, and planets, and did a number of eclipse calculations. In addition to an important treatise on Astronomy, Al-Khwarizmi wrote a book on astronomical tables, which were also translated into European languages and, later, into Chinese.

GEOGRAPHY (14)

In geography, he wrote the book called Kitab Surat al- ard (book of the form of the earth). His works differed from Ptolemy's and he corrected Ptolemy's views in detail. It is a description of a world (known world at that time) map and contains a list of the coordinates of the important places on it. He corrected the distortion that Ptolemy's map had with regard to the length of the Mediterranean. It was much more accurate. However, it failed to replace the Ptolemaic geography used in Europe. He wrote many other books on topics such as clocks, sundials and astrolabes.

Al-Khwarizmi wrote a major work on geography which gives latitudes and longitudes for 2,402 cities and landmarks, forming the basis for a world map. The book, which is based on Ptolemy's Geography, lists with latitudes and longitudes, cities, mountains, seas, islands, geographical regions, and rivers. The manuscript includes maps which on the whole are more accurate than those of Ptolemy.

A number of minor works were written by al-Khwarizmi on topics such as the astrolabe, on which he wrote two works, on the sundial, and on the Jewish calendar. He also wrote a political history containing horoscopes of prominent persons.

Al-Khwarizmi systematized and corrected Ptolemy's research in geography and astronomy/astrology, using his own original findings. He supervised the work of 70 geographers to create a map of the then "known world". Amazingly this map of the "known world" shows the pacific coast of South America-about 700 years before Columbus "discovered" America.

He is also reported to have collaborated in the degree measurements ordered by Khalifah (Caliph) Mamun al-Rashid. These were aimed at measuring of volume and circumference of the earth. His geography book entitled "Kitab Surat-al-Ard," including maps, was also translated. His other contributions include original work related to clocks, sundials and astrolabes. He also wrote Kitab al-Tarikh and Kitab al-Rukhmat (on sundials).

AL KHWARIZMI'S IMPACT ON EUROPE

In 1140 Robert of Chester (who read mathematics in Spain) translated the Arabic title into Latin as Liber algebrae et almucabala, then ultimately gave its name to the discipline of algebra. The Spanish Jew, John of Seville, produced another Latin version.

When his work became known in Europe through Latin translations, his influence made an indelible mark on the development of science in the West: his algebra book introduced that discipline to Europe "unknown till then" and became the standard mathematical text at European universities until the 16^{th} century. In the 16^{th} century it is found in English as algiebar and almachabel and in various other forms but was finally shortened to algebra. He is one of the Muslim scholars who laid the foundations for

Europe's Renaissance and the Scientific Revolution. He also wrote on mechanical devices like the clock, astrolabe, and sundial. His other contributions include tables of trigonometric functions, refinements in the geometric representation of conic sections, and aspects of the calculus of two errors. (15)[[|]]

Several of Al-Khwarizmi's books were translated into Latin in the early 12th century by Adelard of Bath and Gerard of Cremona. The treatises on Arithmetic, Kitab al-Jam'a wal-Tafreeq bil Hisab al-Hindi, and the one on Algebra, Al-Maqala fi Hisab-al Jabr wa-al-Muqabilah, are known only from Latin translations. Introduction of Arabic numerals provided a pivotal advance over the cumbersome Roman numerals. This development of a more convenient number system assisted progress in science, accounting and bookkeeping. Key to this was the use of the number zero, a concept unknown to the West. The use of this number system (Arabic numerals) spread throughout the Muslim world over the next two centuries, assisting the development of science. The Arabic numeral system was first mentioned in Europe around 1200 CE, but Christian adherence to the Roman system hindered its use and introduction. It was only fully accepted in Europe after it was adopted by the Italian traders in the Renaissance of the 16th century, who followed the practice of their Arab trading partners.

MUSLIM IMPACT ON EUROPE (18)

During the Middle Ages the Islamic World had a very significant impact upon Europe, which in turn cleared the way for the Renaissance and the Scientific Revolution. One of the most important of these influences was the study of science.

Ever since Islam was born, Muslims had made immense leaps forward in the area of science. Cities like Baghdad, Damascus, Cairo and Cordoba were the centers of civilization. These cities were flourishing and Muslim scientists made tremendous progress in applied as well as theoretical Science and Technology. In Europe, however, the situation was much different. Europe was in the Dark Ages. It had no infrastructure or central government. To the Muslims, Europe was backward, unorganized, carried no strategic importance and was essentially irrelevant. This considering the time period was in fact true. Nevertheless the Catholic Church (which at the time was the strongest institution in Europe) successfully convinced Christian Europe that the Muslims were infidels. This caused Europeans to think that Muslims were culturally inferior to Europe and thus Europe was unable to benefit from the new scientific discoveries being made in the Islamic lands before the 1100's. By doing this Europe kept itself in the Dark Ages while from China to Spain Islamic Civilization prospered. During the Crusades there was limited contact between Muslims and Christians and not much was transferred. As A. Lewis explains, "The Crusaders were men of action, not men of learning". The real exchange of ideas which led to the Scientific Revolution and to the renaissance occurred in Muslim Spain.

Cordoba was the capital of Muslim Spain. It soon became the center for intellectual enlightenment and learning for all of Europe. Scholars and students from various parts of the world and Europe came to Cordoba to study. The contrast in intellectual activity is demonstrated best by one example: 'In the 9th century, the library of the monastery of St. Gall was the largest in Europe. It boasted 36 volumes. At the same time, that of Cordoba contained over 500,000.

ARABIC MATHEMATICS WORLDWIDE (19)

In the 11th century, the Arab mathematical foundation was one of the strongest in the world. The Muslim mathematicians had invented geometrical algebra and had taken it to advanced levels, capable of solving third and fourth degree equations. The world witnessed a new stage in the development of mathematical science, driven by the numerous translated works from Arabic into European languages.

Unquestionably, Al-Khwarizmi was very influential with his methods on arithmetic and algebra which were translated into much of southern Europe. Again, these translations became popular as algorismi – a term which is derived from the name of Al-Khwarizmi. Not all went smoothly nonetheless. The Arabic numerals introduced by Al-Khwarizmi, like much of new mathematics, were not welcomed wholeheartedly. In fact, in 1299 there was a law in the commercial center of Florence (Italy) forbidding the use of such numerals. Initially, only universities dared use them, but later they became popular with merchants, and eventually became commonly used.

In time, Europe realized the great potential value of the Arab mathematical contributions and put into popular use all that seemed practical. The sciences, with mathematics as their essence, flourished and developed into the disciplines we know today. None would have been the same though, had it not been for that book on restoration, or had the zero not been invented, or had the Arabic numerals not made their way to Europe. That "fondness of science," which inspired an early Arab mathematician to propose calculating by al-jabr and al-muqabala, did much to make the world run as we know it today.

NUMBER ZERO (20)

The 10th millennium saw Muslim mathematical study concentrated in three main sub-disciplines. These were the ongoing progress in algebra, the development of arithmetic algorithms, and the increasing complexity in geometry. In addition, the introduction of the zero was destined to revolutionize mathematics as it allowed for key innovations. It was proposed by Muhammad Bin Ahmad in 967 CE. Zero arrived in the West much later, in 13th century.

In the field of Mathematics the number Zero (0) and the decimal system was introduced to Europe, which became the basis for the Scientific revolution. The Arabic numerals were also transferred to Europe, this made mathematical tasks much easier, problems that took days to solve could now be solved in minutes. The works of Al-Khwarizmi (his Latin name was Alghorismus) were translated into Latin. Al-Khwarizmi (Alghorismus), from whom the mathematical term algorism was derived, wrote Sindh Ind, a compilation of astronomical tables. He, more importantly, laid the ground work for algebra and found methods to deal with complex mathematical problems, such as square roots and complex fractions. He conducted numerous experiments, measured the height of the earth's atmosphere and discovered the principle of the magnifying lens. Many of his books were translated into European languages. Trigonometric work by Alkirmani of Toledo was translated into Latin (from which we get the sine and cosine functions) along with the Greek knowledge of Geometry by Euclid. Along with mathematics, masses of other knowledge in the field of physical science was transferred. (21)

FAMOUS WORKS (22)

1. Al-Jabr wa-al-Muqabilah from whose title came the name "Algebra"
2. Kitab al-Jam'a wal-Tafreeq bil Hisab al-Hindi (on Arithmetic, which survived in a Latin translation but was lost in the original Arabic)
3. Kitab Surat-al-Ard (on geography)
4. Istikhraj Tarikh al-Yahud (about the Jewish calendar)
5. Kitab al-Tarikh
6. Kitab al-Rukhmat (about sun-dials)

BREAKING BOUNDARIES (23)

Certainly, the Renaissance, the Enlightenment and the Industrial Revolution were great achievements – and they occurred mainly in Europe and, later, in America. Yet many of these developments drew on the experience of the rest of the world, rather than being confined within the boundaries of a discrete Western civilization.

Our global civilization is a world heritage – not just a collection of disparate local cultures. When a modern mathematician in Boston invokes an algorithm to solve a difficult computational problem, he/she may not be aware that he/she is helping to commemorate the Arab mathematician Mohammad Ibn Musa-al-Khwarizmi, who flourished in the first half of the 9th century. (The word algorithm is derived from the name al-Khwarizmi.)

THE SQUARE ROOT OF MATH ITSELF (23)

There is a chain of intellectual relations that link Western mathematics and science to a collection of distinctly non-Western practitioners, of whom al-Khwarizmi was one. (The term algebra is derived from the title of his famous book Al-Jabr wa-al-Muqabilah.)

Indeed, al-Khwarizmi is one of many non-Western contributors whose works influenced the European Renaissance and, later, the Enlightenment and the Industrial Revolution. The West must get full credit for the remarkable achievements that occurred in Europe and Europeanized America, but the idea of an immaculate Western conception is an imaginative fantasy.

Modern prosperity, with all its improvement in welfare, has been delivered to humanity by science and technology. In the last two centuries especially, science has delivered better lives for people, longer lives, and for larger populations. The key to unlocking the source of these benefits was scientific method, the relentless search for truth through observation, theorizing and experimentation.

In the 13th century the Muslim world, with its development of the culture of philosophy, science, mathematics, astronomy, physics, chemistry and medicine, led the world. The Muslim world once possessed in its hands the keys to the future prosperity that technology could deliver. Not only that, but with the invention of double entry bookkeeping, it possessed in its hands the blueprint of the plans for the modern corporation. Eventually, after several hundred years, Europe was able to absorb this knowledge and overthrow the dark constraint of its own religion to unlock the mysteries of science and discover the path to prosperity. If the Muslim world had been able to continue on the Qur'anic commands on scientific research, the cause of human progress would have been advanced by about five hundred years. (17)

CONCLUSION

In conclusion, algebra and algorithms are enabling the building of computers, and the creation of encryption. The modern technology industry would not exist without the contributions of Muslim mathematicians like Al-Khwarizmi.

Ms. Carly Fiorina, Hewlett-Packard's Chairman and CEO delivered a speech in Minneapolis, Minnesota on September 26, 2001. The title of her speech was 'TECHNOLOGY, BUSINESS AND OUR WAY OF LIFE: WHAT'S NEXT". (24) She said "There was once a civilization that was the greatest in the world." ……."And this civilization was driven more than anything, by invention. Its architects designed buildings that defied gravity. Its mathematicians created the algebra and algorithms that would enable the building of computers, and the creation of encryption. Its doctors examined the human body, and found new cures for disease. Its astronomers looked into the heavens, named the stars, and paved the way for space travel and exploration." "When other nations were afraid of ideas, this civilization thrived on them, and kept them alive. When censors threatened to wipe out knowledge from past civilizations, this civilization kept the knowledge alive, and passed it on others."

"While modern Western civilization shares many of these traits, the civilization I'm talking about was the Islamic world from the year 800 to 1600, which included the Ottoman Empire and the courts of Baghdad, Damascus and Cairo, and enlightened rulers like Suleiman the Magnificent."

"Although we are often unaware of our indebtedness to this other civilization, its gifts are very much part of our heritage. The technology industry would not exist without the contributions of Arab mathematicians. Sufi poet-philosophers like Rumi challenged our notions of self and truth. Leaders like Suleiman contributed to our notions of tolerance and civic leadership. And perhaps we can learn a lesson from his example: It was leadership based on meritocracy, not inheritance. It was leadership that harnessed the full capabilities of a very diverse population-that included Christianity, Islamic and Jewish traditions."

REFERENCES

1. http://www.sacbee.com/static/archive/news/projects/algebra/121400.html (http://www.sacbee.com/static/archive/news/projects/algebra/121400.html)
2. cstl.syr.edu/fipse/Algebra/Unit1/algebra.htm (cstl.syr.edu/fipse/Algebra/Unit1/algebra.htm)
3. www.mentorproducts.com/what.html (www.mentorproducts.com/what.html)
4. http://www.mathmedia.com/whystudal.html (http://www.mathmedia.com/whystudal.html)
5. http://www.factmonster.com/ce6/sci/A0859533.html (http://www.factmonster.com/ce6/sci/A0859533.html)
6. http://www.infoplease.com/ce6/sci/A0856560.html (http://www.infoplease.com/ce6/sci/A0856560.html)
7. http://members.tripod.com/~khorasan/TajikPersonalities/Khwarizmi.htm (http://members.tripod.com/~khorasan/TajikPersonalities/Khwarizmi.htm)
8. http://www.yorku.ca/sasit/sts/sts3700b/lecture02a.html (http://www.yorku.ca/sasit/sts/sts3700b/lecture02a.html)
9. http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Al-Khwarizmi.html (http://www-gap.dcs.st-and.ac.uk/~history/Mathematicians/Al-Khwarizmi.html)
10. R Rashed, The development of Arabic mathematics: between arithmetic and algebra (London, 1994). www-groups.dcs.st-and.ac.uk/ ~history/Mathematicians/Al-Khwarizmi.html (www-groups.dcs.st-and.ac.uk/ ~history/Mathematicians/Al-Khwarizmi.html)
11. http://www.unhas.ac.id/~rhiza/saintis/khawariz.html (http://www.unhas.ac.id/~rhiza/saintis/khawariz.html)
12. http://www.stetson.edu/~efriedma/periodictable/html/Am.html (http://www.stetson.edu/~efriedma/periodictable/html/Am.html)
13. http://www.ummah.com/forum/archive/index.php/t-4425.html (http://www.ummah.com/forum/archive/index.php/t-4425.html)
14. How Islam Influenced Science, by Macksood Aftab, Managing Editor, The Islamic Herald, March 1995. http://www.themodernreligion.com/science/science_influence.htm (http://www.themodernreligion.com/science/science_influence.htm)
15. IB Syed.: Rise and Fall of Muslims. http://www.irfi.org (http://www.irfi.org)

16. www.ais.org/~bsb/Herald/Previous/95/science.html (www.ais.org/~bsb/Herald/Previous/95/science.html)
17. O'Connor, J. J. and E. F. Robertson. "Arab mathematics: forgotten brilliance." The Mac Tutor History of Mathematics Archive. John J. O'Connor and Edmund F. Robertson. 2001. School of Mathematics and Statistics at University of St. Andrews, Scotland. 2001. http://www-groups.dcs.st-andrews.ac.uk, http://www.alshindagah.com/sepoct2002/arab.html (http://www-groups.dcs.st-andrews.ac.uk, http://www.alshindagah.com/sepoct2002/arab.html)
18. How Islam Influenced Science, by Macksood Aftab, Managing Editor The Islamic Herald, March 1995. http://www.geocities.com/WestHollywood/Park/6443/Science/science_influence.htm (http://www.geocities.com/WestHollywood/Park/6443/Science/science_influence.htm)
19. http://www.answers.com/topic/al-khwarizmi (http://www.answers.com/topic/al-khwarizmi)
20. Does Globalization Equal Westernization? By Amartya Sen. The Globalist, Monday, March 25, 2002 http://www.theglobalist.com/DBWeb/StoryId.aspx?StoryId=2353 (http://www.theglobalist.com/DBWeb/StoryId.aspx?StoryId=2353)
21. CARLY FIORINA, Minneapolis, Minnesota , September 26, 2001 "Technology, Business and our Way of Life: What's Next?" http://www.hp.com/hpinfo/execteam/speeches/fiorina/minnesota01.html (http://www.hp.com/hpinfo/execteam/speeches/fiorina/minnesota01.html)

Comments are closed.